Name: ______________________________

Instructor: __________________________

Math 10170, Practice Exam 2
April 8, 2016

- The Honor Code is in effect for this examination. All work is to be your own.
- You may use your Calculator.
- The exam lasts for 50 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 15 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	______
8.	______
9.	______
10.	______
11.	______
Total	______

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Multiple Choice

1.(6 pts.) The probability distribution of a random variable X given below, find $E(X)$ and the standard deviation $\sigma(X)$.

k	Pr(X = k)	XP(X)
-1	$\frac{1}{2}$	$-\frac{1}{2} = \frac{8}{16}$
0	$\frac{1}{8}$	0
1	$\frac{1}{16}$	$\frac{1}{16}$
2	$\frac{1}{16}$	$\frac{2}{16}$
3	$\frac{1}{4}$	$\frac{3}{4} = \frac{12}{16}$
	$\mu =$	$E(X) = \frac{7}{16}$

k	Pr(X = k)	$X - \mu$	$(X - \mu)^2$	$(X - \mu)^2 P(X)$
-1	$\frac{1}{2}$	$\frac{-23}{16}$	$\frac{529}{256}$	$\frac{529}{512}$
0	$\frac{1}{8}$	$\frac{-7}{16}$	$\frac{49}{256}$	$\frac{49}{2048}$
1	$\frac{1}{16}$	$\frac{9}{16}$	$\frac{81}{256}$	$\frac{81}{4096}$
2	$\frac{1}{16}$	$\frac{25}{16}$	$\frac{625}{256}$	$\frac{625}{4096}$
3	$\frac{1}{4}$	$\frac{41}{16}$	$\frac{1681}{256}$	$\frac{1681}{1024}$
			$\sigma^2 =$	$\frac{735}{256}$

$$\boxed{\sigma = \frac{7\sqrt{15}}{16}.}$$

(a) $-\frac{1}{4}$ (b) $\frac{1}{16}$ (c) $\frac{7}{16}$ (d) $\frac{1}{4}$ (e) $-\frac{3}{8}$

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2.(6 pts.) A basketball player with career field goal percentage of 80% takes 100 shots in a row. If his probability of making a basket on every shot is 0.8, what is the longest run of misses that you would expect in the data?

The probability of a miss is 0.2. Therefore the longest run of misses in 100 shots has length approximately equal to

$$-\frac{\ln((1-0.2)100)}{\ln(0.2)} \approx 2.72 \approx 3.$$

(a) 13

(b) 3

(c) 10

(d) 6

(e) 20

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3.(6 pts.) Given a sequence with two values, success (S) and failure (F), with N_s success' and N_f failures, let X denote the number of runs (of both S's and F's). Wald and Wolfowitz determined that for a random sequence of length N with N_s success' and N_f failures (note that $N = N_s + N_f$), the number of runs has mean and standard deviation given by

$$E(X) = \mu = \frac{2N_s N_f}{N} + 1, \qquad \sigma(X) = \sqrt{\frac{(\mu - 1)(\mu - 2)}{N - 1}}.$$

The distribution of X is approximately normal if N_s and N_f are both bigger than 10.

What is the value of X for the following sequence of success' and failures and what is its Z-score (the number of standard deviations from the mean)?

$$S\ F\ S\ S\ S\ F\ F\ S\ S\ S\ S\ S\ F\ F\ F\ F\ S\ S\ F\ S$$

We have $N = 20$, $N_s = 12$ and $N_f = 8$. The observed value of X = number of runs in the sequence is $X = 9$. We have

$$E(X) = \mu = \frac{2(12)(8)}{20} + 1 = 10.6, \qquad \sigma(X) = \sqrt{\frac{9.6)(8.6)}{19}} \approx 2.08.$$

Recall that the Z-Score for an observation x of a random variable X is $Z = \dfrac{x - E(X)}{\sigma(X)}$. In this case the Z-score is

$$Z = \frac{9 - 10.6}{2.08} \approx -0.769 \approx -0.77.$$

(a) $X = 9, Z \approx 0.77$

(b) $X = 5, Z \approx 2.69$

(c) $X = 5, Z \approx -2.69$

(d) $X = 9, Z \approx -0.77$

(e) $X = 10, Z \approx 0.29$

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4.(6 pts.) Statistics collected over a five year period for performance of wide receivers in the vertical jump at the NFL combine showed an average of 35.1 inches and a standard deviation of 3.2. Assuming that the distribution of the observations was mound shaped (roughly normally distributed), roughly what percentage of the wide receivers had a vertical jump height between 31.9 inches and 38.3 inches.

The z-score of 31.9 is $\frac{31.9 - 35.1}{3.2} = -1$ and the z-score of 38.3 is $\frac{38.3 - 35.1}{3.2} = 1$. Therefore roughly 68% of wide receivers had a vertical jump height between 31.9 inches and 38.3 inches.

(a) 99.7% (b) 68% (c) 95%

(d) 50% (e) 80%

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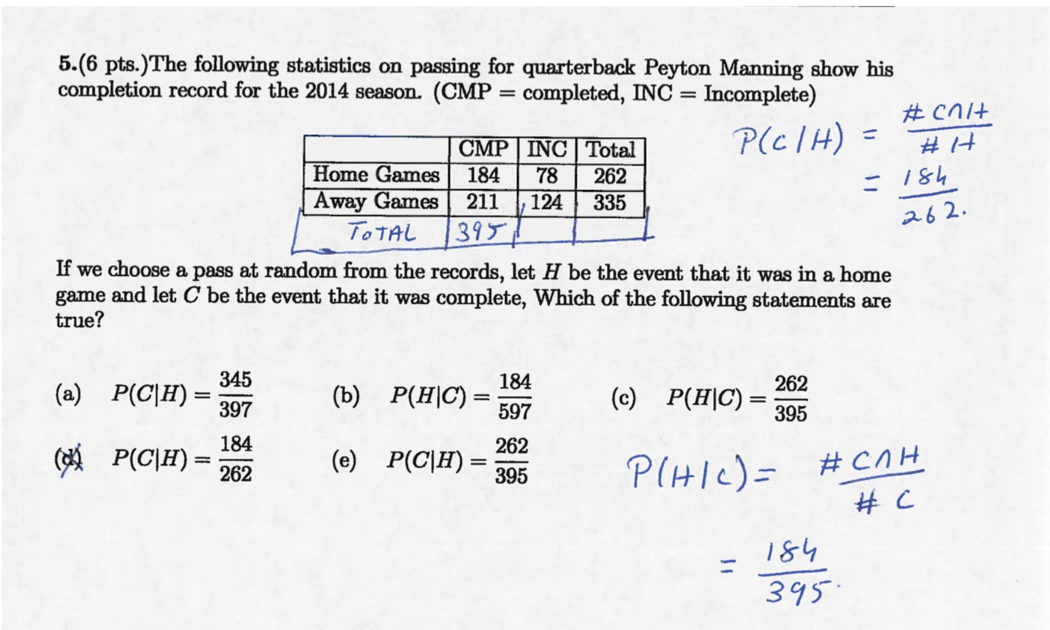
5.(6 pts.)The following statistics on passing for quarterback Peyton Manning show his completion record for the 2014 season. (CMP = completed, INC = Incomplete)

	CMP	INC	Total
Home Games	184	78	262
Away Games	211	124	335



5.(6 pts.)The following statistics on passing for quarterback Peyton Manning show his completion record for the 2014 season. (CMP = completed, INC = Incomplete)

	CMP	INC	Total
Home Games	184	78	262
Away Games	211	124	335
TOTAL	395		

$P(C|H) = \frac{\# C \cap H}{\# H} = \frac{184}{262}$.

If we choose a pass at random from the records, let H be the event that it was in a home game and let C be the event that it was complete, Which of the following statements are true?

(a) $P(C|H) = \frac{345}{397}$ (b) $P(H|C) = \frac{184}{597}$ (c) $P(H|C) = \frac{262}{395}$

(d) $P(C|H) = \frac{184}{262}$ (e) $P(C|H) = \frac{262}{395}$

$P(H|C) = \frac{\# C \cap H}{\# C} = \frac{184}{395}$.

If we choose a pass at random from the records, let H be the event that it was in a home game and let C be the event that it was complete, Which of the following statements are true?

(a) $P(C|H) = \frac{184}{262}$ (b) $P(H|C) = \frac{184}{597}$ (c) $P(C|H) = \frac{262}{395}$

(d) $P(C|H) = \frac{345}{397}$ (e) $P(H|C) = \frac{262}{395}$

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6.(6 pts.) In a simplified model of a tennis serve, the server must decide whether to serve to the receiver's forehand (F) or to the backhand (B). The receiver must anticipate whether the serve will come to the forehand (F) or the backhand (B). For players $Robert(R)$ and $Carl(C)$, it is estimated that

- if Robert serves to the forehand (F) and Carl anticipates correctly, then there is a 50% chance that Robert will win the serve.
- On the other hand if Carl does not correctly anticipate the serve to the forehand, there is a 70% chance that Robert will win the serve.
- if Robert serves to the backhand (B) and Carl anticipates correctly, then there is a 40% chance that Robert will win the serve.
- On the other hand if Carl does not correctly anticipate the serve to the backhand, there is a 60% chance that Robert will win the serve.

Which of the following shows the correct payoff matrix for Robert for this constant sum game?

7.(6 pts.) In a simplified model of a tennis serve, the server must decide whether to serve to the receiver's forehand (F) or to the backhand (B). The receiver must anticipate whether the serve will come to the forehand (F) or the backhand (B). For players $Robert(R)$ and $Carl(C)$, it is estimated that

- if Robert serves to the forehand (F) and Carl anticipates correctly, then there is a 50% chance that Robert will win the serve. FF → 50
- On the other hand if Carl does not correctly anticipate the serve to the forehand, there is a 70% chance that Robert will win the serve. FB → 70
- if Robert serves to the backhand (B) and Carl anticipates correctly, then there is a 40% chance that Robert will win the serve. BB → 40
- On the other hand if Carl does not correctly anticipate the serve to the backhand, there is a 60% chance that Robert will win the serve. BF → 60

Which of the following shows the correct payoff matrix for Robert for this constant sum game?

(a)

Robert \ Carl	F	B
F	50	40
B	70	60

(b)

Robert \ Carl	F	B
F	40	60
B	50	70

(c)

Robert \ Carl	F	B
F	50	70
B	60	40

(d)

Robert \ Carl	F	B
F	50	30
B	60	40

(e)

Robert \ Carl	F	B
F	70	50
B	60	40

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(a)

		Carl F	Carl B
Robert	F	70	50
Robert	B	60	40

(b)

		Carl F	Carl B
Robert	F	50	70
Robert	B	60	40

(c)

		Carl F	Carl B
Robert	F	50	40
Robert	B	70	60

(d)

		Carl F	Carl B
Robert	F	40	60
Robert	B	50	70

(e)

		Carl F	Carl B
Robert	F	50	30
Robert	B	60	40

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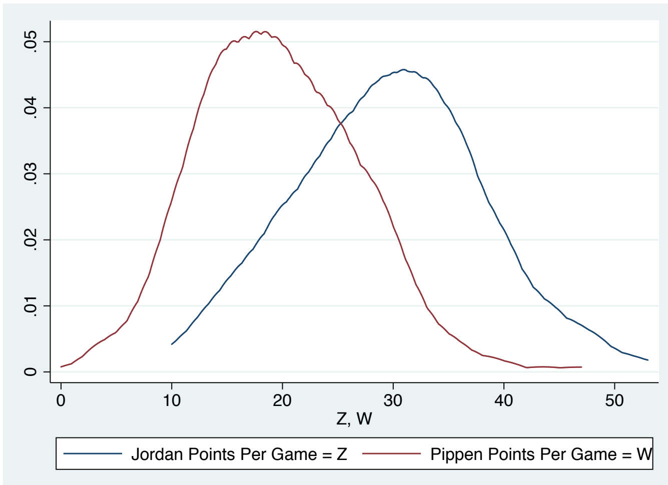
7.(6 pts.) The following graphs show the probability distributions of points scored per game played for Michael Jordan and Scottie Pippen.



Which of the following statements is true?

The proportion of games played by either player with a score less than (greater than) than 30 is the area under that players probability distribution to the left(right resp.) of 30, (the total area under each distribution is 1).

(a) The proportion of games played by Michael Jordan in which he scored more than 30 points was greater than 0.5.

(b) The proportion of games played by Michael Jordan in which he scored more than 30 points was approximately equal to 0.04.

(c) The average number of points scored per game for both players was the same.

(d) The proportion of games played by Scottie Pippen in which he scored more than 30 points was greater than 0.5.

(e) The proportion of games played by Scottie Pippen in which he scored less than 30 points was approximately equal to 0.02.

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Partial Credit

You must show your work on the partial credit problems to receive credit!

8.(14 pts.) The following statistics on field goal attempts by LeBron James were collected from 52 games of Basketball played this season:

	FGM	FGNM	Total = FGA
Home Games	233	151	384
Away Games	283	227	510

Here
FGM means Field Goals Made;
FGNM means Field Goals attempted but not Made;
FGA means Field Goals Attempted.

(a) Use these statistics to estimate the probability that LeBron James will make the next field goal he attempts given that it is a home game.

$$P(FG|H) = \frac{n(FG \cap H)}{n(H)} = \frac{233}{384} \approx 0.6008.$$

(b) Use your estimate from part (a) to estimate the probability that LeBron James will make his first three attempts at field goal in his next home game, given that all of his shots at field goal are independent of each other.

We use the prior information that the game is a home game to estimate his probability of a field goal on each shot at 0.6008. We also use independence to calculate the probability:

$$P(\text{FG on 1st} \cap \text{FG on 2nd} \cap \text{FG on 3rd}) = P(\text{FG on 1st})P(\text{FG on 2nd})P(\text{FG on 3rd})$$
$$= 0.6008^3 \approx 0.2234.$$

(c) Are the events "Making a Field Goal" and "Playing in a Home Game" Independent? (Justify your answer)

We can rephrase this question as "Is $P(FG|H) = P(FG)$?" or "Is $\frac{233}{384} = \frac{516}{894}$? or "Is $0.6068 = 0.577$?" The answer is no, the events are not independent. However, with more statistical tools, we would consider a margin of error.

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9.(12 pts.) On a reality TV show

- A contestant must complete a 2 step obstacle course to get to the next round.
- At the beginning of each stage of the obstacle course, the judge flips a fair coin to decide which task the contestant should complete.
- In the first stage of the obstacle course the contestant must either eat a bag of live worms or a bag of habanero peppers.
- If the contestant completes stage 1 of the obstacle course, they get to attempt stage two, otherwise, they are out of the competition.
- In the second stage of the obstacle course the contestant must either retrieve a stick from a tank of alligators or a ball from a pit of snakes.
- If the contestant completes stage 1 of the obstacle course, they get to move on to round two, otherwise, they are out of the competition.

Sydney has a 50% chance of finishing a bag of worms and a 20% chance of finishing a bag of habaneros. She has a 30% chance of retrieving a stick from a tank of alligators(and living on to tell the tale) and a 20% chance of getting a ball out of a pit of snakes (without being poisoned).

Use a tree diagram to determine the probability that Sydney will make it to the next round of the competition.

Using the tree diagram on the next page, we see that there are four paths leading to completion of both stages. We multiply the probabilities along these paths to find the probability of the path. We then add the four resulting probabilities to get that the probability that Sydney will make it to the next round is:

$(0.5)^3(0.3)+(0.5)^3(0.2)+(0.5)^2(0.2)^2+(0.5)^2(0.3)(0.2) = 0.0375+0.025+0.01+0.015 = 0.0875.$

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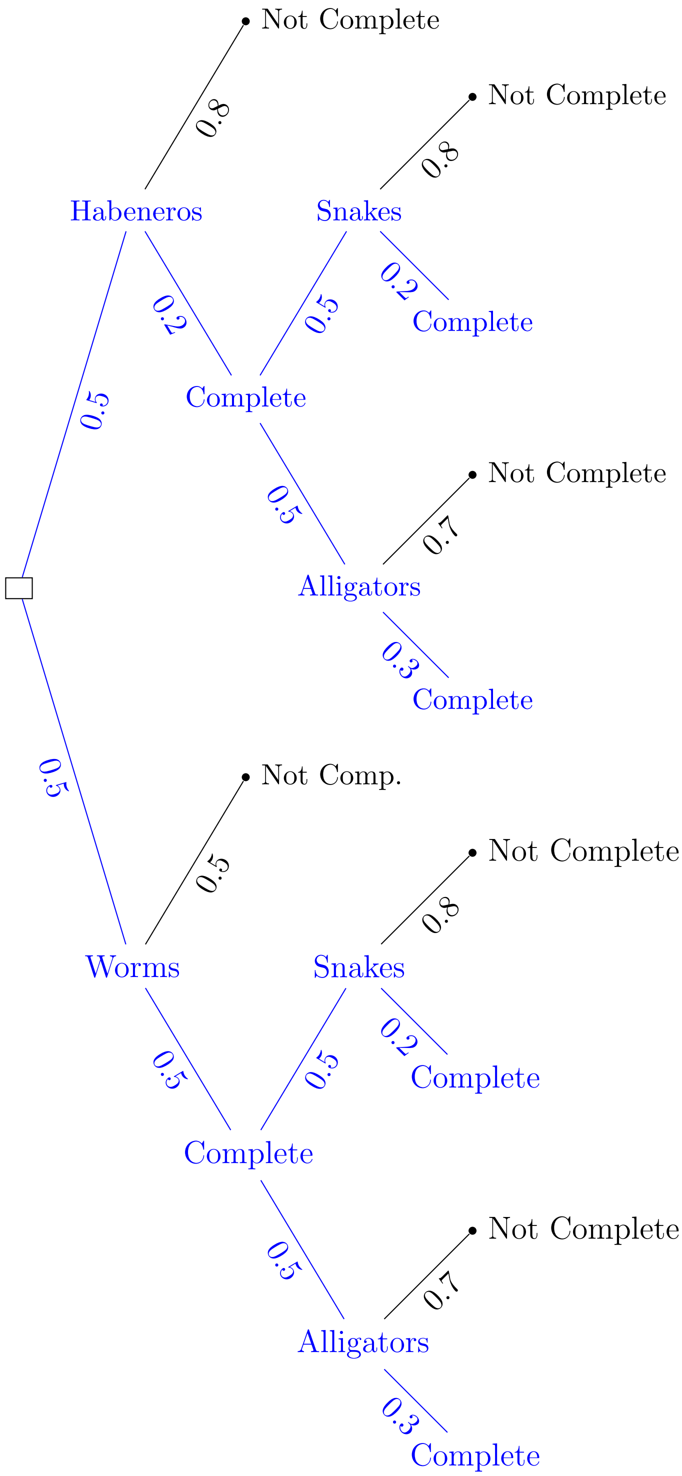

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10.(12 pts.) The following shows data for 50 consecutive passes for quarterback Drew Brees show whether each pass was complete(C) or incomplete (I):

C C C C I C C C I C I I C I C C C C C C C C C C I

I I C C I C C C I C I I C C C I C C C C C C C C C

(a) How many runs (of C's and I's) are there in the data?

(b) If X denotes the number of runs in a randomly generated sequence of C's and I's of length N with N_C C's and N_I I's, X has an approximately normal distribution with

$$E(X) = \frac{2N_C N_I}{N} + 1, \quad \text{and} \quad \sigma(X) = \sqrt{\frac{(\mu - 1)(\mu - 2)}{N - 1}}.$$

Applying this distribution to the case given above, what is the Z score of the above observed set of data?

(c) Using your knowledge of the empirical rule, would you say that the above data was randomly generated or not? Justify your answer.

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Partial Credit

You must show your work on the partial credit problems to receive credit!

8.(12 pts.) The following shows data for 50 consecutive passes for quarterback Drew Brees show whether each pass was complete(C) or incomplete (I):

C C C C | I | C C C | I | C | I I | C | I | C C C C C C C C C C | I
I I | C C | I | C C C | I | C | I I | C C C | I | C C C C C C C C C |

(a) How many runs (of C's and I's) are there in the data?

19 Runs

(b) If X denotes the number of runs in a randomly generated sequence of C's and I's of length N with N_C C's and N_I I's, X has an approximately normal distribution with

$$E(X) = \frac{2N_C N_I}{N} + 1, \quad \text{and} \quad \sigma(X) = \sqrt{\frac{(\mu - 1)(\mu - 2)}{N - 1}}.$$

Applying this distribution to the case given above, what is the Z score of the above observed set of data?

$N = 50$

$N_C = \# C\text{'s} = 37$

$N_I = \# I\text{'s} = 13$

$E(X) = \frac{2(37)(13)}{50} + 1 = 20.24 = \mu$

$\sigma(X) = \sqrt{\frac{(19.24)(18.24)}{49}} \cong 2.67$

Z-score of our observation of $X \to 19$

$= \frac{19 - \mu}{\sigma} = \frac{19 - 20.24}{2.67} = -0.4644$

(c) Using your knowledge of the empirical rule, would you say that the above data was randomly generated or not? Justify your answer.

If our observation of X was more than 2 standard deviations away from E(X), we would question whether the data was randomly generated (with the same probability of C throughout). However no red flags are raised here since the observed value of X (19) is within one standard deviation of the mean (|Z-score| < 1).

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11.(20 pts.) Take Home Part

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2.	(a)	(•)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(•)	(e)
4.	(a)	(•)	(c)	(d)	(e)
5.	(•)	(b)	(c)	(d)	(e)
6.	(a)	(•)	(c)	(d)	(e)
7.	(•)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice ______

8. ______

9. ______

10. ______

11. ______

Total ______